Work, energy and power

Unit 5

Contents

You have met the concepts of energy and work before in Grade 9. An understanding of energy is essential to many things you use every day.

5.1 Work as a scalar product

By the end of this section you should be able to:

- Differentiate between energy, work and force.

KEY WORDS

work *the product of displacement and the force in the direction of the displacement. It is measured in joules.*

You covered the concept of work in Grade 9. You do **work** whenever you move a displacement when exerting a force in the direction of the displacement. Work done is defined as the magnitude of the force exerted in the direction of the displacement (or distance moved) multiplied by the displacement.

Remember that the unit of energy is the joule.

1 joule = 1 newton × 1 metre

So 1 joule is the work done when a force of 1 newton moves through a distance of 1 metre. This is the definition of the joule.

Here is a worked example to remind you.

Worked example 5.1

The girl in Figure 5.1 is lifting a heavy box onto a table. She uses a force of 200 N. The top of the table is 1.2 m above the floor. How much work does she do?

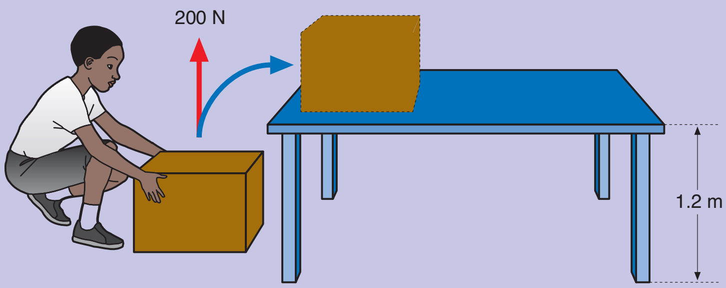

Figure 5.1 *A girl lifting a heavy load*

We know the force and the distance moved, so we can calculate the work done:

work done = force × distance moved by the force

= 200 N × 1.2 m = 240 J

So the girl does 240 J of work. We can say also say that 240 J of energy has been transferred from the girl to the box.

The box has gained 240 J of potential energy because it is higher up than it was before.

Activity 5.1

In small groups, discuss the difference between carrying a box down a corridor and pushing the same box. What work is done in each case?

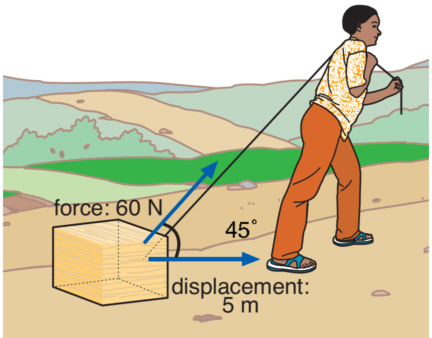

Figure 5.2 *How much work is this man doing?*

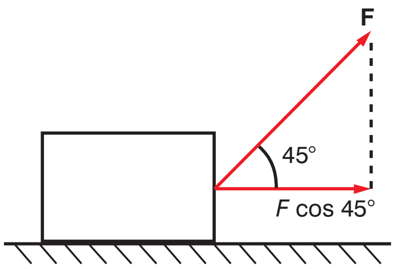

Figure 5.3

In the worked example on page 99, the girl is doing work against the force of gravity.

Both force and displacement moved are vectors.
Work done is the scalar product of force and displacement:

$$W = \mathbf{F} \bullet \mathbf{d} = Fd \cos \theta$$

where F and d are the magnitudes of the vectors.

Work done is a scalar – it does not have directional properties.

Look at Figure 5.2. We can calculate the work done by the man in moving the box.

We can show the forces on a free body diagram (Figure 5.3).

The force in the direction of the displacement is $F \cos 45°$, so work done is

$$W = Fd \cos \theta$$

which is the equation for the scalar product of the force and displacement vectors. In Unit 2, you learnt that the scalar product of two vectors can also be expressed as:

$$\mathbf{a} \bullet \mathbf{b} = a_x b_x + a_y b_y$$

where the vectors are given in component form and are $\mathbf{a} = \begin{bmatrix} a_x \\ a_y \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_x \\ b_y \end{bmatrix}$.

Worked example 5.2

A box is attached to a rope, which runs over a pulley above the box. The man pulls the rope with a force of 75 N at an angle of 50° to the horizontal. The man raises the box 1.5 m. Use the scalar product to find the work done on the box.

Draw a free body diagram to show the forces (Figure 5.4).

In component form, the force is: $\mathbf{F} = \begin{bmatrix} 48.21 \\ 57.45 \end{bmatrix}$ and the displacement is $\mathbf{d} = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}$.

The work done is:

$W = \mathbf{F} \bullet \mathbf{d} = (48.21 \times 0) + (57.45 \times 1.5) = 0 + 86.175 = 86.175$ J

Alternatively you could work out the work done using the formula involving the angle between the vectors. Figure 5.4 shows that this angle is 40°. So the work done is:

$W = Fd \cos 40°$

$= 75 \text{ N} \times 1.5 \text{ m} \times \cos 40°$

$= 75 \text{ N} \times 1.5 \text{ m} \times 0.766$

$= 86.175$ J

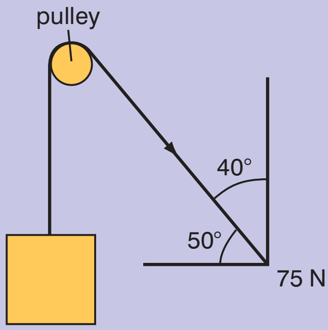

Figure 5.4

In this worked example, note that the angle between the vectors is 40°, as shown in Figure 5.4.

Summary

In this section you have learnt that:

- Work done is the scalar product of the force and displacement vectors.

Review questions

1. A water container is dragged up a slope. The displacement is $\begin{bmatrix} 5 \\ 1 \end{bmatrix}$ metres and the force used is $\begin{bmatrix} 50 \\ 25 \end{bmatrix}$ N.

   What is the total work done?

2. A man drags a box $\begin{bmatrix} 20 \\ -1 \end{bmatrix}$ m with a force of $\begin{bmatrix} 35 \\ 25 \end{bmatrix}$ N.

   How much work does the man do on the box?

5.2 Work done by a constant and variable force

By the end of this section you should be able to:

- Describe and explain the exchange among potential energy, kinetic energy and internal energy for simple mechanical systems, such as a pendulum, a roller coaster, a spring, a freely falling object.

Work is a term that you will already be familiar with both in everyday use and from your earlier studies of physics. The way we use the word 'work' in physics is not the same as the way we use it in everyday life. We may say, for example, that it is hard work to sit at a desk and read this book, but in the physics sense no work is being done.

If we are to do work in the physics sense, two things must happen. They are:

- we must exert a force
- while exerting the force, we must move a distance in the direction of the force.

These conditions are met when we slide a piece of furniture across a rough floor (we do work against the opposing force of friction), when we lift a heavy object off the ground (doing work against the opposing force of gravity) or when we stretch a spring (doing work against the tension in the spring as it tries to contract again).

Activity 5.2

You are going to pull a block up a slope and find the work done in pulling the block up the slope.

- Arrange a wooden board to form a slope at an angle of about 30° to the horizontal.
- Place a block of wood on the slope and attach a newtonmeter.
- Pull the block slowly up the slope and record the force needed to pull the block. Try to pull the block with a constant force – you may need to practise a couple of times.
- Measure the length of the slope.
- Calculate the work done in moving the block up the slope.
- Also record the vertical height of the slope and the mass of the block – you will need this for Activity 5.7 on page 115.

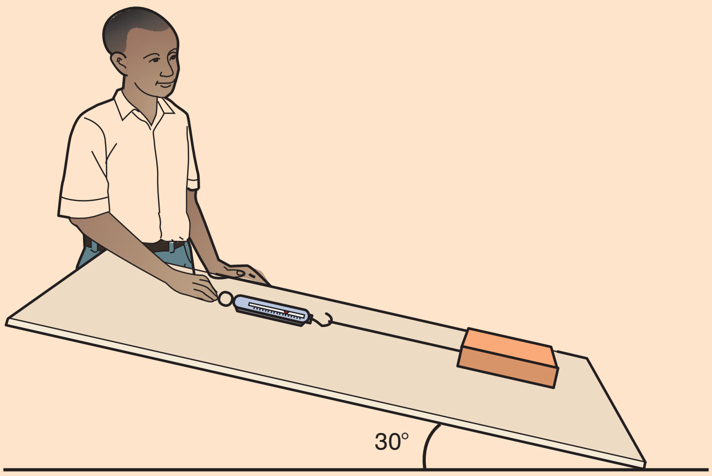

***Figure* 5.5** *Doing work: pulling a block up a slope*

Variable force

So far we have only dealt with constant forces. In practice many forces are not constant. For example, when a driver changes the position of an accelerator pedal in a vehicle, the driving force of the engine is changed. Drivers do not keep the accelerator pedal in the same position but vary it – so the force being applied is not constant.

We cannot use the equation work done = force × distance moved when the force varies – this equation needs the force to be constant. However, if we can find the average force used, then we can calculate the amount of work done using the equation.

In Unit 2, you learnt that when you plot a graph of velocity against time, the area under the graph is the displacement. The same principle applies when we draw a graph of force against displacement.

When a force is constant, the graph will be a horizontal straight line, as shown in Figure 5.6. The work done is 5 N × 10 m = 50 N m.

When a force varies, we cannot use the equation work done = force × distance moved. But the relationship for the area under the graph is still true. If we are able to record the force used and the displacement and plot a graph, we could find the work done by finding the area under the graph, as shown in Figure 5.7.

You could find an approximation to the area by breaking it down into thin vertical strips and finding the area of each strip. However, if you know the average force or can find an estimate of it, you can estimate how much work has been done. The work done is then the average force multiplied by the displacement.

For example, look at Figure 5.7. You can estimate the average force by putting a ruler on top of the graph as though you were going to draw a horizontal line. Adjust the position of the ruler so that the area between the graph line and the ruler is about the same above the ruler as it is below the ruler – this will give you an estimate of the average force.

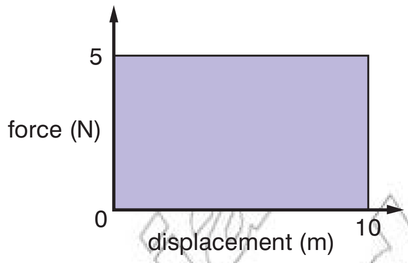

Figure 5.6 *Graph of force against displacement for a constant force*

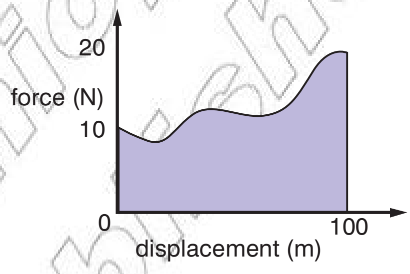

Figure 5.7 *Graph of force against displacement for a variable force*

Summary

In this section you have learnt that:

- Work, energy and force are different things.
- When a force varies, you can calculate the work done directly if you know the average force exerted.
- Work done by a force can be calculated from the area under a graph of force against displacement.

Review question

1. Imagine that you have dragged a heavy box 100 m. Draw a graph of force against distance to show how the force might vary with distance.

5.3 Kinetic energy and the work–energy theorem

By the end of this section you should be able to:

- Identify the relationship between work and change in kinetic energy.
- Analyse and explain common situations involving work and energy, using the work–energy theorem.

Energy is not an easy concept. You may associate energy with motion, but not all forms of energy involve motion. For example, potential energy is based on the position or configuration of an

KEY WORDS

energy *the amount of work that can be performed by a force; it is a scalar quantity and is measured in joules* object. You can measure most forces that cause work to be done and you can measure the displacement of the object that was caused by the force, but you cannot measure the energy directly. You need to multiply together the two quantities that you can measure (force and displacement) to find the work done or energy.

In Grade 9 you learnt about **kinetic energy** and the work–energy theorem. All moving objects have kinetic energy – the amount of energy is related to the velocity of the object. Kinetic energy is given by the equation:

$$E_k = \tfrac{1}{2}mv^2$$

where E_k is the kinetic energy of the object, m is the mass of the object and v is the magnitude of the velocity of the object. Even though velocity is a vector, kinetic energy is a scalar quantity.

The work–energy theorem states that if the kinetic energy of an object changes because of a force acting on the body, the mechanical work is the difference in kinetic energy. It is given by the equation:

$$\mathbf{W = \Delta E_k = E_{k2} - E_{k1} = \tfrac{1}{2}m(v_2^2 - v_1^2)}$$

where W is the work done, ΔE_k is the change in kinetic energy, E_{k2} and E_{k1} are the kinetic energies after and before the force acts, respectively, and v_2 and v_1 are the velocities of the body after and before the force acts.

We can derive the work–energy theorem from Newton's second law of motion. We will use the direction of the force as a frame of reference, as shown in Figure 5.8. A force F acts on a body of mass m over a distance s. At displacement 0, the velocity is v_1 and at displacement s, the velocity is v_2.

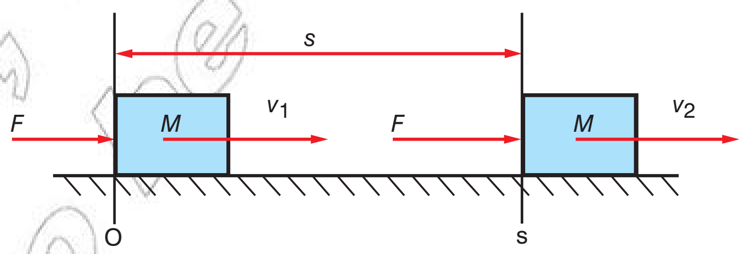

***Figure 5.8** A constant force acting on a body over a distance s*

According to Newton's second law of motion:

$$F = ma$$

We can use the equation of motion $v^2 = u^2 + 2as$ to find an expression for a and substitute this into the equation for Newton's second law:

$$v_2^2 = v_1^2 + 2as$$

$$2as = v_2^2 - v_1^2$$

$$a = (v_2^2 - v_1^2)/2s$$

So the force acting on the body is:

$$F = m(v_2^2 - v_1^2)/2s$$

KEY WORDS

kinetic energy *the energy that a moving object has. The faster an object is moving, the more energy it has.*

REMEMBER

Remember that friction is not a form of energy – it is a force. You have to exert a force to overcome the static friction and then the kinetic friction once the object is moving. So you have to do work against the force, and work is done to achieve this.

Discussion activity

In small groups discuss the following questions.

- What are positive work and negative work?
- What effect do they have on the kinetic energy of a body?
- What forms can energy take and how can it be transformed between these different forms?

Report your conclusions back to the rest of the class.

Now the work done on the body is:

$$W = Fs = \frac{m(v_2^{\,2} - v_1^{\,2})}{2s} \times s = \tfrac{1}{2}mv_2^{\,2} - \tfrac{1}{2}mv_1^{\,2}$$

Figure 5.9 *Energy is transferred to the ball when it is thrown.*

Worked example 5.3

A boy throws a ball. The ball leaves the boy's hand with a velocity of 16 m/s and has a mass of 200 g.

Assuming that the boy's hand moved through 1.2 m when throwing the ball, what is the average force the boy applied to the ball?

As the ball was at rest, the work done on the ball is the kinetic energy of the ball, according to the work–energy theorem.

Kinetic energy of ball, $E_k = \tfrac{1}{2}mv^2$

$E_k = Fd$

So $F = E_k/d = \tfrac{1}{2}mv^2/d$

Substituting the values into the equation:

$F = \tfrac{1}{2} \times 0.2\ \text{kg} \times 16^2\ \text{m}^2/\text{s}^2\ /1.2\ \text{m} = 21.3\ \text{N}$

In the worked example above, the action of the force by the boy's hand on the ball transferred energy from the boy to the ball (Figure 5.9).

Worked example 5.4

A car of mass 1250 kg is travelling at a velocity of 15 m/s due East. The driver applies the brakes to slow the car down to a velocity of 3 m/s due East.

a) What is the work done in slowing the car down?

b) Assume that the car took 3 seconds to slow down. What was the force of the brakes?

First draw a diagram to show the velocities (Figure 5.10).

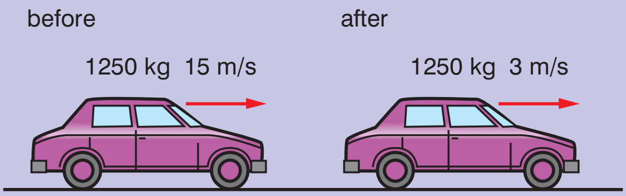

Figure 5.10

a) Using the equation above:

$W = \tfrac{1}{2}m(v_2^{\,2} - v_1^{\,2})$

Substituting in the values:

$W = \tfrac{1}{2} \times 1250(3^2 - 15^2) = \tfrac{1}{2} \times 1250 \times -215$

$= -135\ 000\ \text{J}$

b) Use Newton's second law:

$F = ma$

Work out the acceleration from the initial and final velocities and the time taken for the change in speed:

$a = (v - u)/t$

So $F = m\ (v - u)/t$

Substituting in the values:

$F = 1250\ \text{kg} \times (3\ \text{m/s} - 15\ \text{m/s}) \div 3\ \text{s}$

$= 1250\ \text{kg} \times -12\ \text{m/s} \div 3\ \text{s}$

$= -5000\ \text{N}$

The minus sign means that the force is acting to the left, opposite to the direction the car is travelling in.

In the worked example 5.4, as the car is being slowed down, the acceleration is in a direction opposite to that in which the car is travelling. This also means that the force is in the opposite direction and so negative work is done, because the mechanical energy of the car has decreased.

Also, it was assumed that the car was travelling along a flat road. If the road was sloping, the mechanical energy of the car would also have been changing because of the force due to gravity as well as the force from the brakes.

When this book is sitting on a table, the force of gravity is acting on the book and there is a force of mg being exerted upwards by the table on the book. But no work is done by the table on the book because no energy is transferred into or out of the book.

Project: mousetrap car

You are going to make a toy car powered by a mousetrap that will go the greatest distance, have the highest speed or highest average speed.

In Grade 9 you made a car using a mousetrap – you built a car that was powered only by the spring of a mousetrap (Figure 5.11). Here you are going to have another go.

Can you improve your design so that your car does better?

Can you make your car go the greatest distance, have the highest speed or highest average speed?

Remember that your car may not work well at first. You will need to test your car to see how well it works – you will then probably want to make improvements to your design.

Basic design

The car makes use of the spring in the mousetrap. You attach one end of a piece of string to the 'snapper' part of the mousetrap. The other end of the string is attached to the drive axle of your car by a hook. You then wind the string round the drive axle by turning the wheels backwards so that the snapper arm is pulled back towards the axle (Figure 5.12).

When the drive wheels are released, the string is pulled off the drive axle by snapper moving forwards because of the spring of the mousetrap.

If you decide that you want to go for the highest speed, you need the energy stored in the spring to be released quickly. If you want to go for the longest distance, you need the energy stored in the spring to be released slowly.

The record for the longest distance a mousetrap car has travelled is over 180 metres!

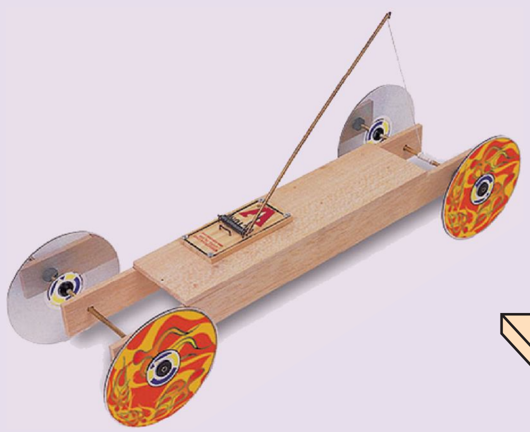

Figure 5.11 *A mousetrap car*

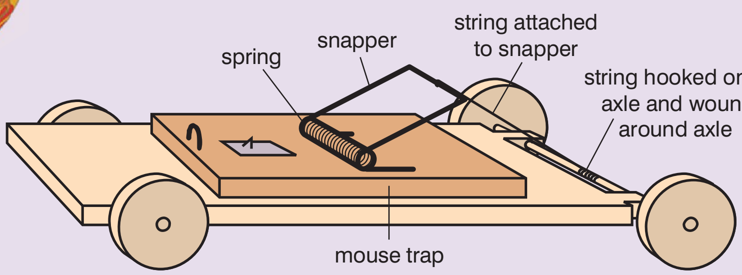

Figure 5.12 *Principle of how a mousetrap car works*

Summary

In this section you have learnt that:

- The work–energy theorem can be derived from Newton's second law of motion.

Review questions

1. A football of mass 550 g is at rest on the ground. The football is kicked with a force of 108 N. The footballer's boot is in contact with the ball for 0.3 m.
   a) What is the kinetic energy of the ball?
   b) What is the ball's velocity at the moment it loses contact with the footballer's boot?
2. A car of mass 1200 kg accelerates from 5 m/s to 15 m/s. The force of the engine acting on the car is 6000 N.
   Over what distance did the force act?

5.4 Potential energy

By the end of this section you should be able to:

- Determine the energy stored in a spring.
- Describe and explain the exchange among potential energy, kinetic energy and internal energy for simple mechanical systems, such as a pendulum, a roller coaster, a spring, a freely falling object.

You do work when you push or pull something against a force that is opposing you. When you stop pulling or pushing, does the system tend to fly back to its original position if you let it? If so, then at least some of the energy you have fed in must have been stored as potential energy. **Potential energy** tends to get released as the system returns to its original state. Some examples of where energy is stored as potential energy are gravitational potential energy and elastic potential energy.

KEY WORDS

potential energy *the energy possessed by an object because of its position or configuration. Its units are joules.*

When you raise something from the floor to a high shelf, the object gains gravitational potential energy. You have forced the object and the Earth apart against their gravitational attraction. If you let them, the two will pull each other back together – in other words, the object will fall back to the ground.

When you compress or stretch a spring or bend a bow to fire an arrow, energy is stored as elastic potential energy.

When you lift something, you do work on it against the force of gravitational attraction. The energy gained by the object is the force multiplied by the distance moved. We can express this in a slightly

Discussion activity

How can you use the fact that work is the scalar product of force and displacement to explain that no work is done when you carry a box at the same height, but work is done if you change the object's height?

Can you also use this to explain negative work?

Worked example 5.5

A boy climbs a flight of stairs. His mass is 65 kg and the displacement is $\begin{bmatrix} 4 \\ 4 \end{bmatrix}$ m.

How much potential energy has he gained?

Vertical distance is 4 m.

Gain in gravitational potential energy

$= 65 \text{ kg} \times 9.8 \text{ m/s}^2 \times 4 \text{ m}$

$= 2548 \text{ J}$

Discussion activity

How does your choice of frame of reference affect the amount of gravitational energy an object appears to have?

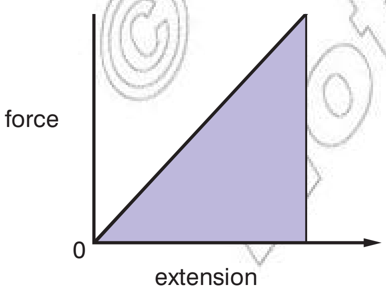

Figure 5.14

different way. The force is the mass of the object multiplied by the acceleration due to gravity. The distance moved is the increase in height.

So work done, $W = mg\Delta h$

The work done is energy and we say that the object has gained gravitational potential energy. So we can say that the gain in gravitational potential energy is given by the equation:

$$\text{GPE} = mg\Delta h$$

Activity 5.3

You are going to find out how much a spring stretches when masses are hung on it. Use different masses and measure the extension.

The basic set up for the apparatus is shown in Figure 5.13.

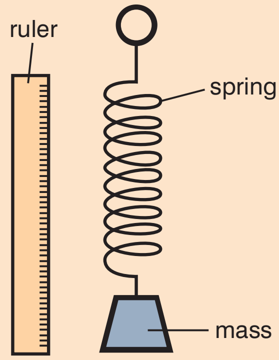

Figure 5.13

Plan your experiment.

What are you going to measure? How will you measure it?

What will you do to make your results more reliable?

Can you find a relationship between the length of the spring and the mass that you hang on the end of it?

Make a prediction of what the length of the spring will be when you hang a certain mass on it. Hang the mass on the spring and see if your prediction is correct.

Write a report of your experiment using the writing frame in Unit 1.

You should have found from Activity 5.3 that there is a linear relationship between the force on the spring and the extension of the spring. The expression is:

$$\mathbf{F} = -k\mathbf{x}$$

where F is the force on the spring, x is the extension of the spring and k is the spring constant.

Figure 5.14 shows a graph of the force on a spring against the extension of the spring. Extension is the same as distance – in Section 5.2 we saw that the area under the graph is the work done. In Figure 5.14, this is the amount of work done on the spring to stretch it by a certain distance. This is also the amount of energy stored in the spring.

So the energy stored in a spring can be written as the area of the triangle which is

$$E = \tfrac{1}{2}Fx$$

From before $F = kx$, so

$$E = \tfrac{1}{2}kx \times x = \tfrac{1}{2}kx^2$$

Worked example 5.6

A mass of 125 g is attached to a horizontal spring that sits on a frictionless surface. One end of the spring is attached to a block that is massive. The spring is stretched by 10 cm and the spring constant is 200 N/m.

a) How much energy is stored in the spring?

b) The mass is let go. What is the highest velocity of the mass?

a) First draw a diagram to show all of the variables.

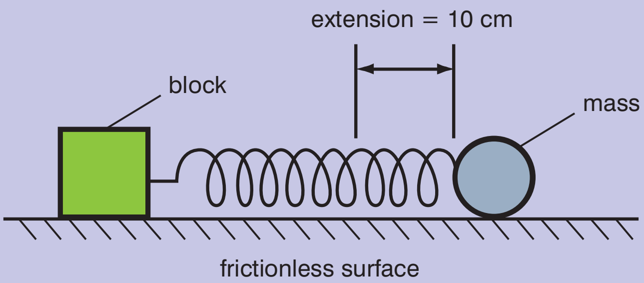

Figure 5.15

The energy stored in the spring is given by the equation

$E = \tfrac{1}{2}kx^2$

Substituting in the values:

$E = \tfrac{1}{2} \times 200\ \text{N/m} \times (0.1\ \text{m})^2$

$= 1\ \text{J}$

b) When the mass is let go, the spring will contract and the mass will move.

When the mass is moving the fastest, all of the stored energy will have been transferred to kinetic energy.

Use the equation $E = \tfrac{1}{2}mv^2$

So $v^2 = 2E/m$

and $v = \sqrt{(2E/m)}$

Substituting in the values:

$v = \sqrt{(2 \times 1\ \text{J} \div 0.125\ \text{kg})}$

$= \sqrt{(16\ \text{m}^2/\text{s}^2)}$

$= 4\ \text{m/s}$

Activity 5.4

1. Describe the energy changes that take place as a roller coaster goes from its starting position, up to the top of the first slope, then down the slope and round a loop-the-loop.

   Will the roller-coaster keep all its mechanical energy? Explain why or why not.

2. A spring is set up with a mass hanging from it like you used in Activity 5.3. The mass is pulled down and then let go. The mass oscillates up and down.

   Describe the energy changes that are taking place in the mass and spring as it oscillates.

Summary

In this section you have learnt that:

- Gravitational potential energy is given by the equation GPE = $mg\Delta h$, where Δh is the change in height.
- The extension x of a spring is given by the equation $F = kx$, where F is the force and k is the spring constant.
- The energy stored in a spring is given by the equation $E = \frac{1}{2}kx^2$, where k is the spring constant and x is the extension.
- In an oscillation, energy is transferred between potential energy and kinetic energy.

Review question

1. a) A boy walks up a hill. His displacement from his starting point is (800, 150) m.

   How much gravitational potential energy has he gained?

   b) The boy then walks to a village. The displacement from his starting point is (400, –50) m.

   How much gravitational potential energy did he lose going from the top of the hill to the village?

   c) What was the boy's net change in gravitational potential energy from his starting point to the village?

2. A spring has a spring constant of 75 N/m. It is stretched by 20 cm.

   How much energy is stored in the spring?

3. A force of 40 N is used to stretch a spring which has a spring constant of 350 N/m.

   How much energy is stored in the spring?

4. A spring has a spring constant of 150 N/m and a mass is 100 g is attached to it. The spring sits on a horizontal frictionless surface and the other end of the spring is attached to a solid block.

   The mass is pulled by 10 cm to stretch the spring and then let go.

   What is the highest velocity of the mass?

5.5 Conservation of energy

By the end of this section you should be able to:

- Predict velocities, heights and spring compressions based on energy conservation.
- Apply the law of mechanical energy conservation in daily life situations.
- Describe and explain the exchange among potential energy, kinetic energy and internal energy for simple mechanical systems, such as a pendulum, a roller coaster, a spring, a freely falling object.
- Solve problems involving conservation of energy in simple systems with various sources of potential energy, such as springs.

In Grade 9, you learnt about the law of conservation of energy, which states that energy cannot be created or destroyed. It can only be changed from one form to another.

In a system the mechanical energy of the system stays constant unless there is a force such as friction acting on the system. The total mechanical energy is:

$$E = U + E_K$$

where E is the total mechanical energy, E_K is the kinetic energy and U is the potential energy. The potential energy can be gravitational potential energy or energy stored in a spring, for example.

When a spring is stretched, work is done because a force has been used to move one end of the spring by a certain displacement. Work is also done against gravity when you walk up stairs and you gain gravitational potential energy. When you walk down stairs, work is done by gravity and you lose gravitational potential energy.

We can show this as:

Work done against gravity, $W = \Delta U$

Work done by gravity, $W = -\Delta U$

In Unit 3, we used the equations of motion to solve the problem of how high a ball will go before falling back to earth when it is thrown into the air. We can use the law of conservation of energy to help us solve the same problem.

When a ball is thrown into the air, all of its energy is kinetic energy. As the vertical component of the ball's velocity decreases, its total kinetic energy decreases. Some of the kinetic energy is transferred to gravitational potential energy. When the ball reaches its maximum height, all of the kinetic energy from the vertical component of the ball's velocity has been transferred to potential energy.

Activity 5.5: Energy changes in a pendulum bob

You aare going to explore the energy changes in a pendulum bob as it swings to and fro.

Set up a pendulum by attaching it securely to a support bar. Mount a metre stick horizontally, as shown in Figure 5.16. Hold the pendulum bob to one side as shown and let the bob go.

What happens to the bob? Where does the bob reach on the other side?

What is happening to the energy of the bob as it swings backwards and forwards?

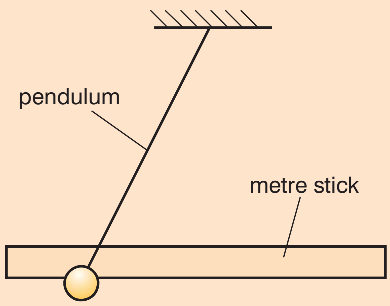

Figure 5.16 *Pendulum for Activity 5.5*

Worked example 5.7

A ball is thrown with a velocity of 20 m/s at an angle of 20° to the horizontal. The mass of the ball is 200 g.

What is the maximum height reached by the ball?

Draw a diagram to show the ball's path (Figure 5.17).

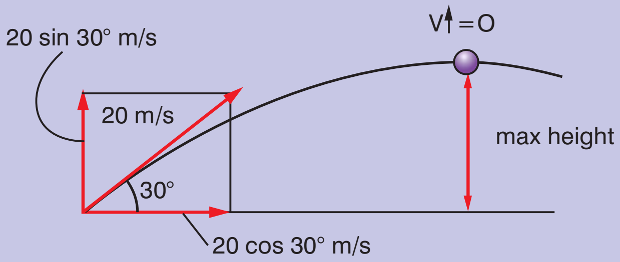

Figure 5.17

Resolve the velocity of the ball into horizontal and vertical components.

$$\mathbf{v} = \begin{bmatrix} 20 \cos 30° \\ 20 \sin 30° \end{bmatrix}$$

When the ball is thrown, the kinetic energy of the vertical component of its velocity is:

$KE = ½mv_y^2$

At the maximum height, all of this energy is transferred to gravitational potential energy:

$GPE = mg\,\Delta h$

So $mg\Delta h = ½mv_y^2$

$\Delta h = ½v_y^2/g$

Substituting in the values:

$\Delta h = ½(20 \sin 30°)^2/9.8$

$= 5.1$ m

Worked example 5.8

A 2 kg mass hangs from a piece of rope of length 3 m. It is pulled out on one side a perpendicular distance of 0.5 m and let go.

What is the velocity of the mass as it passes through the lowest point?

First draw a diagram (Figure 5.18).

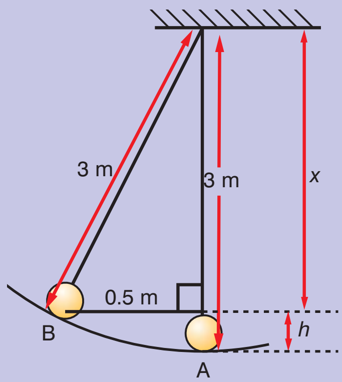

Figure 5.18

The lowest point is when the mass is at A. We need to work out the height that the mass falls during its swing. This is the height of B above A.

Using Pythagoras' theorem, $x^2 = 9 - 0.25 = 8.75$

So $h = 3 - \sqrt{8.75}$

Gravitational potential energy at B = mgh

Kinetic energy at A = $½mv^2$

As the mechanical energy of the system does not change (we can ignore the effects of air resistance):

$½mv^2 = mgh$

$v^2 = 2gh$

Substituting the values into the equation:

$v^2 = 2 \times 9.8 \times (3 - \sqrt{8.75})\ m^2/s^2$

$= 0.822\ m^2/s^2$

So $v = \sqrt{0.822}$ m/s = 0.91 m/s

Collisions

In Unit 4 you learnt about elastic and inelastic collisions and that momentum is conserved in all collisions. Kinetic energy is not conserved in all collisions. If the collision is elastic, kinetic energy is conserved. If the collision is inelastic, kinetic energy has not been conserved.

Worked example 5.9

Two roads are perpendicular and meet at a junction.

Car A of mass 1000 kg travels along one road at 20 m/s due North.

Car B of mass 1300 kg, travels due West along the other road at 16 m/s.

At the junction the cars collide and move together with a velocity of $\begin{bmatrix} 9.04 \\ 8.70 \end{bmatrix}$ m/s.

Is the collision elastic or inelastic?

We need to work out the kinetic energies of the cars before and after collision.

KE of car A = $\frac{1}{2}mv^2 = \frac{1}{2} \times 1000 \times 20^2$

= 200 000 J

KE of car B = $\frac{1}{2} \times 1300 \times 16^2$ = 166 400 J

Total KE of cars before collision
= 200 000 + 166 400 = 366 400 J

Use Pythagoras to find the magnitude of velocity of two cars after collision:

$v^2 = 9.04^2 + 8.70^2$

KE of two cars after collision

$= \frac{1}{2} \times (1000 + 1300) \times (9.04^2 + 8.70^2)$

= 181 023 J

The collision is inelastic because kinetic energy has not been conserved.

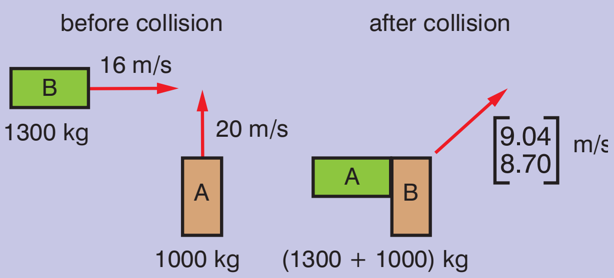

Figure 5.19

Summary

In this section you have learnt that:

- The law of conservation of energy states that energy cannot be created or destroyed.
- The law of conservation of energy can be use to solve problems involving kinetic and potential energy.
- Kinetic energy is conserved in an elastic collision, but not in an inelastic collision.

Review questions

1. A ball of mass 500 g is kicked into the air at an angle of 45°. It reaches a height of 12 m.

   What was its initial velocity?

2. A pendulum bob has a mass of 1 kg. The length of the pendulum is 2 m. The bob is pulled to one side to an angle of 10° from the vertical.
   a) What is the velocity of the pendulum bob as it swings through its lowest point?
   b) What is the angular velocity of the pendulum bob?

3. A pool ball of mass 100 g and velocity $\begin{bmatrix}5\\-4\end{bmatrix}$ m/s collides with a stationary pool ball of the same mass. After the collision, the pool balls have velocities $\begin{bmatrix}2\\-3\end{bmatrix}$ m/s and $\begin{bmatrix}3\\-1\end{bmatrix}$ m/s. Is the collision elastic? Explain your answer.

5.6 Conservative and dissipative forces

By the end of this section you should be able to:

- Distinguish between conservative and non-conservative forces.
- Analyse situations involving the concepts of mechanical energy and its transformation into other forms of energy according to the law of conservation of energy.

Activity 5.6

One student picks up a book from a desk and places it directly on a high shelf. A second student picks up a book from the same desk, walks away briefly from the desk before putting it on the high shelf. A third student picks up a book from the same desk, walks around the classroom several times and then places it on the high shelf.

Which book has had more work done on it? Why?

Discuss in small groups and report back to the rest of the class.

The mechanical energy of a system is the sum of the potential energy and the kinetic energy of the system. Some forces cause mechanical energy to be lost from a system, whereas others do not.

Look at Figure 5.20a. Suppose you pick up the pen from the table and move it through the air. You then put the pen back on the table in the same place. What can you say about the work done on the pen? It is zero (we are assuming that there is no friction from the air). The force on the pen is the gravitational attraction of the Earth. The pen gained some gravitational potential energy when it was lifted up off the table. It then lost this gravitational potential energy when it was put back on the table.

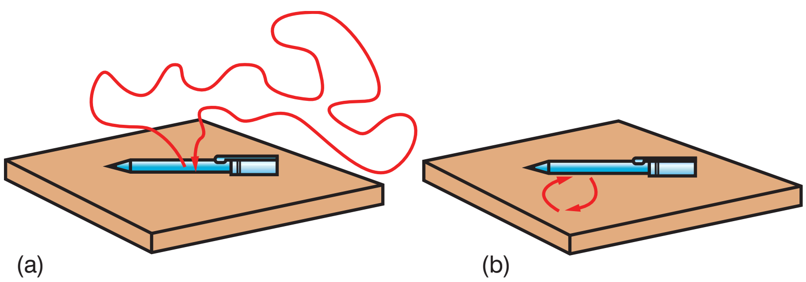

Figure 5.20 *Conservative (a) and dissipative (b) forces*

Look at Figure 5.20b. Suppose you push the pen across the table and then push it back again to its original position. In this case work has been done on the pen, because you had to apply a force to push the pen across the table. The force is due to friction between the table and the pen.

The first example is an example of a **conservative force**. Gravity is an example of a conservative force. It does not matter which path you take – it is the vertical distance that determines whether you have gained or lost gravitational potential energy. If you return to the same place, the net change in your gravitational potential energy is zero. A spring is another example of something that exerts a conservative force.

The second example showed a **dissipative force**. When you push the pen across the table, you have to do work to overcome the frictional force. When you return the pen to its original place, there has been some work done on the pen, and it has lost mechanical energy because of the frictional force. Another example of a dissipative force is drag caused by the air.

In the absence of friction, mechanical energy is conserved. When friction is present, mechanical energy is not conserved and the mechanical energy lost is equal to the work done against friction.

We can express this in the following relationship:

$$W_{nc} = E_f - E_i$$

where W_{nc} is the work done by non-conservative forces, E_f is the final energy of the system and E_i is the initial energy of the system.

When a block is dragged over a surface, W_{nc} is the work done by a force in overcoming the friction of the surface to move the block through a displacement *d*.

Summary

In this section you have learnt that:

- When a conservative force acts on a body, the work done is independent of the path taken by the body.
- Springs and gravity are examples of conservative forces.
- A dissipative force causes mechanical energy to be lost from a body when it is moving.
- Friction is an example of a dissipative force.

Review question

1. What are the differences between conservative and dissipative forces?

KEY WORDS

conservative force *a force that does no work when a body moves on a closed path*

dissipative force *a force that does work when a body moves on a closed path*

Activity 5.7

Look at your results from Activity 5.2 earlier in this unit.

- Work out how much gravitational potential energy the block will have gained at the top of the slope.
- What was the work done in moving the block?
- Why is this different to the gain in gravitational potential energy?
- What else can you work out from your results?

Discussion activity

What examples of conservative and dissipative forces can you think of? Justify why you think each force is conservative or dissipative.

KEY WORDS

power *the rate at which work is done or energy is expended. Its units are watts.*

Worked example 5.10

An engine raises a load of 100 kg from a mine that is 300 m deep in 2 minutes.

What is the power of the engine?

The force needed to lift the load is:

force = weight of load = 100 kg × 9.8 m/s^2 = 980 N

Calculate the work done in raising the load:

work done = force × distance moved = 980 N × 300 m = 294 000 N

Calculate the power:

$$\text{power} = \frac{\text{work done}}{\text{time taken}}$$

time taken = 2 × 60 = 120 seconds

power = 294 000/120 = 2450 W

Project work

Investigate what the major source of energy used in houses in your area is. Is there more than one main source?

Write a report on your findings.

5.7 Power

By the end of this section you should be able to:

- Define and work out power.

In Grade 9, you learnt about **power**. As for work and energy, the term power as has a different meaning in physics to its meaning in everyday life. We can describe a truck as powerful, but we really mean that it is capable of exerting a large force.

Power is the rate at which work is done, or the work done per second. It is measured in the units joules per second (j/s), which are also called watts (W).

$$\text{power} = \frac{\text{total work done}}{\text{total time taken}}$$

Activity 5.8

You are going to find the power of your arm muscles.

You do work when you lift an object, because you have to move the box upwards – you are doing work against gravity. The faster you move the box, the greater your power.

- Find the mass of the object.
- Hold the object in one hand and put your arm down by your side.
- Lift the weight up above your head so that your arm is stretched vertically above your head.
- Drop your arm by your side.
- Repeat nine times.
- Use a stopwatch to measure the time it takes you to raise and lower the object ten times.
- Measure the vertical distance you move the object through.
- Calculate the work done.
- Calculate your power $\left(= \frac{\text{work done}}{\text{time taken}}\right)$
- What time should you use to work out your power? Why do you think this?

Summary

In this section you have learnt that:

- Power = work done ÷ total time taken.

Review questions

1. A weightlifter lifts 200 kg through 1.8 m in 2 s.
   a) What is the weightlifter's power?
   b) Why is his actual power likely to be higher than this?
2. A petrol engine raises 200 litres of water in a well from a depth of 7 m in 6 seconds. Show that the power of the engine is about 2330 W.
3. Look at question 1 on page 101 . It takes 4 seconds to drag the container up the slope. What is the power?
4. Look at question 2 on page 101. The man takes 12 seconds to drag the box. What is his power?
5. A spring with a spring constant of 275 N/m is stretched 20 cm in 2 seconds.

   What is the power applied to stretch the spring?

End of unit questions

1. Construct a glossary of all the key terms in this unit. You could add it to the one you made for Units 1–4.
2. Figure 5.21 shows a smooth bowl XYZ. A ball is released from point X.
   a) Describe how the ball will move.
   b) What energy changes occur as the ball moves?
   c) How would your answers to parts a) and b) be difference if the surface of the ball was rough?

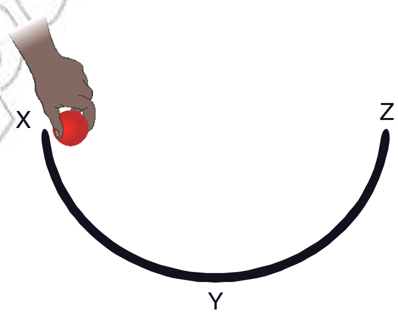

Figure 5.21

3. Two children with the same mass climb a ladder to the top of a slide. One of them goes down the slide and returns to the base of the ladder. The other child comes back down the ladder.

   Has each child lost the same amount of energy? Explain your answer.
4. What are the differences between work, energy and force?
5. What energy changes take place in a stone that falls from a cliff?
6. Dahnay carried a box (5, 4) m. The box had a mass of 5 kg.

   Dahnay says that over 300 J of work was done on the box. Is Dahnay correct? Explain your answer.
7. How can you find the work done from a graph of force against displacement?
8. How can you derive the work–energy theorem form Newton's second law of motion?